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LETTER TO THE EDITOR

The resistance of two quantum point contacts in series

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Abstract. The resistance of two quantum point contacts (QPCs) in series is investigated experimentally. The voltage is measured both across and in between the series pair. The normalised transmission coefficient, $\bar{T}$, for ballistic transport is determined from the data using simple theory. $\bar{T}$ varies between 0.5 and 1.0 depending on the number of conducting 1D channels through each point contact. $\bar{T}$ is always found to be a minimum value when the number of 1D channels in each QPC is the same.

The conductance of a single quantum point contact (QPC) is known to be quantised approximately in units of $2e^2/h$. The conductance is given by $N(2e^2/h)$ where N is the number of one-dimensional conducting channels in the contact. This remarkable result was discovered independently by two groups [1, 2]. It is important because it provides direct evidence for the existence of one-dimensional states when a two-dimensional electron gas (2DEG) is laterally confined and also because the quantisation can occur in zero magnetic field. An equally interesting result was also discussed by Wharam and co-workers [3] who presented evidence that the total resistances of two QPCs in series was not the sum of the individual resistances but was the larger of the two resistances. They considered this to be the case even when the two QPCs were separated by a region of unrestricted 2DEG. The interpretation of this result involves the idea of adiabatic transport in which the electrons from the first point contact traverse the 2DEG region while preserving their one-dimensional quantum numbers. An equivalent description is that the transmission probability to the second QPC for an electron emerging from the first QPC is unity. Such adiabatic transport has been definitely observed by van Wees and co-workers [4] in high magnetic fields ($B > 2.1$ T). However, in their experiment, the situation was somewhat different because the magnetic field was sufficiently high that Landau levels were well established in the bulk material and transport through the QPCs was via edge states that were automatically collimated. The adiabatic transport hypothesis has been considered in more detail theoretically by Beenakker and van Houten [5]. They predict that the total resistance of a series pair of QPCs will depend in a quite complicated fashion on the detailed shape of the confinement potentials. Only in very special cases, with a high degree of collimation of the electrons, do they predict that adiabatic transport will occur in zero magnetic field. We present, in this Letter, a

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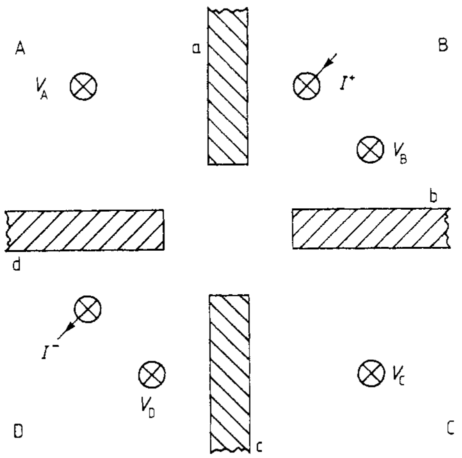

Figure 1. A schematic diagram of the gates, labelled a, b, c and d, and the electrical contacts used in the experiment. Opposite gates are 0.48 μm apart.

set of experiments on two QPCs in series in which we are able to measure not only the total voltage drop across the two QPCs but also the intermediate voltage between them. By using the Landauer–Büttiker [6, 7] formula we are able to translate these voltages into a transmission coefficient for ballistic transport. Beenakker and van Houten [5] have considered theoretically a similar situation and calculate the voltage across a pair of identical QPCs. However, they do not consider the intermediate voltage.

The central region of our structure is shown schematically in figure 1. The 2DEG is formed in an n-type GaAs/(AlGa)As heterostructure with a carrier concentration of 3.4×10^{15} m^{-2} and mobility at 4.2 K of approximately 100 m^2 V^{-1} s^{-1} after illumination, corresponding to an elastic mean free path of $\sim$10 μm.

The independently variable gates are fabricated by electron beam lithography and lift-off techniques from Ti/Au. The lithographic width of each gate is 0.15 μm and the distance between the ends of opposite gates is 0.48 μm.

We label the four gates a, b, c, d and the four 2DEG regions A, B, C, D as illustrated. Each of the 2DEG regions has two ohmic contacts (Au/Ge/Ni). We use the notation that the QPC formed by applying the same negative voltage V_g(ab) to gates a and b (and no bias to the other pair of gates) has a conductance G_{ab} etc. Measurements were made at a temperature of 70 mK using a four-wire resistance bridge with 30 μV excitation.

A typical variation of $(G_{cd})^{-1}$ with gate voltage is shown in the lowest curve (curve A) of figure 2. Equivalent curves are found for all pairs of gates. Assuming that the Fermi wavelength λ_F in the channel is unaffected by gate voltage, we can estimate the channel width using the relation $w = N\lambda_F/2$. The gate voltages at which the quantised steps occur then provide the width, w_{ij}, of the channel defined by gates i and j according to the following relation:

$$w_{ij}\ (\mathrm{nm}) = 210(\pm 5)\ V_g(\mathrm{V}) + 410(\pm 10) \qquad V_g < 0,$$

where V_g is the applied gate voltage. Since the channels only begin to be defined for $V_g < -1$ V, the maximum channel widths compare reasonably well with the lithographic width. The width w_{ad}, however, does *not* obey this formula. Instead,

$$w_{ad}\ (\mathrm{nm}) = 138\ V_g\ (\mathrm{V}) + 400$$

although again the channel begins to be defined at < -1 V. Presumably this difference is due to some artefact of fabrication. However, it turns out to be a fortunate accident

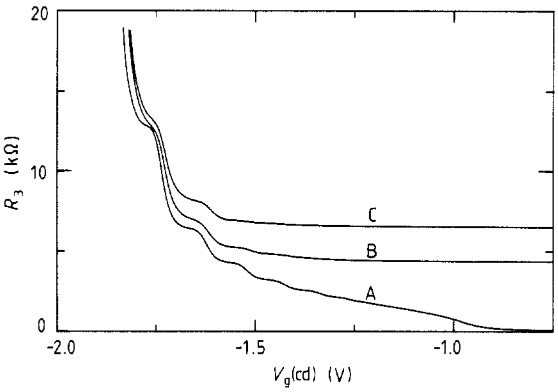

Figure 2. Series resistance, R_3, as a function of the gate voltage applied to gates c and d. The gate voltage on a and b is held constant at 0 V (curve A), −1.61 V (curve B) and −1.77 V (curve C). Curve A also corresponds to $G_{\rm CD}^{-1}$ since a and b are unbiassed.

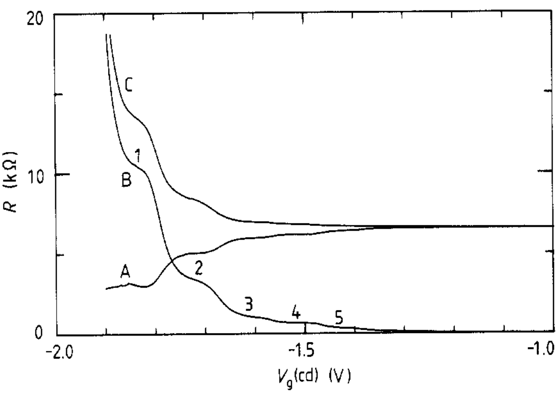

Figure 3. R_1, R_2 and R_3 plotted as a function of the gate voltage on c and d with the voltage on gates a and b held constant at −1.77 V. Curve A, R_1; curve B, R_2, curve C, R_3. Each resistance plateau in R_2 is labelled by $N_{\rm cd} = (h/2e^2)G_{\rm cd}$.

in that for most of the measurements we pass the current between B and D and we are able to use A as the intermediate voltage contact. Even when $G_{\rm ab}$ and $G_{\rm cd}$ are pinched off, $G_{\rm ad}$ will be open.

In curves B and C of figure 2, we show the resistance of a series pair of QPCs. For each of these curves the bias on gates c and d is varied while the bias on gates a and b is held constant at 0 V in curve A, −1.6 V in curve B, and −1.77 V in curve C. We define the series resistance as $R_3 = (V_{\rm B} - V_{\rm D})/I$ where the current is flowing between regions B and D. The lowest curve, therefore, is the resistance of the QPC defined by c and d with no other constraint. Since the gates do not begin to pinch-off until the gate voltage $V_g < -1$ V, the series resistance of the pair for gate voltage between −1 V and 0 V is determined entirely by $G_{\rm ab}$ for the other curves. Figure 2 shows very clearly that there is a substantial but not complete ballistic contribution to the series resistance.

The extra flexibility of the measurement of the intermediate voltage is illustrated in figure 3. Here we plot three resistances; R_1 defined as $(V_{\rm B} - V_{\rm A})/I$, R_2 defined as $(V_{\rm A} - V_{\rm D})I$ and R_3, the series resistance, is $(V_{\rm B} - V_{\rm D})/I$. In each case the current, I, is passing between B and D. The conditions for figure 3 are that the bias on gates a and b is held constant at −1.77 V so that $G_{\rm ab}^{-1} = 6493\ \Omega$, only 1% larger than the quantised value, $h/4e^2$. Then the bias on gates c and d is swept. Classically R_1 would be the (constant) resistance $G_{\rm ab}^{-1}$, R_2 would be $G_{\rm cd}^{-1}$ and $R_3 = G_{\rm ab}^{-1} + G_{\rm cd}^{-1}$. This is clearly not the case in our experiment, and is illustrated most emphatically by R_1 *falling* as the increasingly negative bias applied to gates c and d starts to narrow the c–d channel. The plateaux which occur in R_1, R_2 and R_3 are clear but not at quantised values. This behaviour can be understood by a simple analysis of the experiment in terms of a transmission coefficient for ballistic transport.

We define the three conductances $G_1 = R_1^{-1}$, $G_2 = R_2^{-1}$ and $G_3 = R_3^{-1}$. Then it follows that

$$G_1 = eI/(\mu_B - \mu_A) \qquad G_2 = eI/(\mu_A - \mu_D) \qquad G_3 = eI/(\mu_B - \mu_D). \tag{1}$$

where μ_α is the chemical potential in the region α (α = A, B, C or D). Following Beenakker and van Houten [5] we apply the Büttiker formula [6] for the current I_α in the lead α with N_α quantum channels,

$$(h/2e)I_\alpha = (N_\alpha - R_\alpha)\mu_\alpha - \sum_{\alpha \neq \beta} T_{\beta\alpha}\mu_\beta \tag{2}$$

where R_α and $T_{\beta\alpha}$ are related to the reflection coefficient back into the reservoir α and the transmission coefficient from reservoir β to reservoir α respectively. In the simple case of two terminals, we can take $\mu_\beta = 0$ and equation (2) reduces to

$$G_{\alpha\beta} = eI_\alpha/\mu_\alpha = (2e^2/h)(N_\alpha - R_\alpha) \simeq (2e^2/h)N \tag{3}$$

which is the approximate quantisation obtained for a single QPC, with N an integer.

For the situation with two QPCs in series, we define for convenience $\mu_A = 0$. Noting that G_{ab} and G_{cd} as previously defined are equivalent to G_{AB} and G_{AD} in the formalism of (3), we obtain from (2)

$$I = (G_{ab}/e)\,\mu_B - (2e/h)T\mu_D \tag{4a}$$

$$-I = (G_{cd}/e)\,\mu_D - (2e/h)T\mu_B \tag{4b}$$

$$0 = -T_{DA}\mu_D - T_{BA}\mu_B \tag{4c}$$

where we have taken A to be a voltage contact with the current flowing between B and D. T is proportional to the direct, ballistic, transmission probability through both channels. T_{DA} and T_{BA} are proportional to the transmission probabilities for electrons scattering from D and B respectively into the 2DEG region between the two QPCs. Normalisation of transmission probabilities gives

$$T + T_{BA} = (h/2e^2)G_{ab} \tag{5a}$$

$$T + T_{DA} = (h/2e^2)G_{cd} \tag{5b}$$

where, for totally adiabatic transport only

$$(2e^2/h)T = \mathrm{Min}\,(G_{ab}, G_{cd}). \tag{6}$$

Equation (6) ensures that in the pure ballistic case the total conductance is governed by the smaller of the two conductances. Simple manipulation of equations (4) and (5) gives

$$G_1 = G_{ab} + (2e^2/h)T[G_{ab} - (2e^2/h)T]/[G_{cd} - (2e^2/h)T] \tag{7a}$$

$$G_2 = G_{cd} + (2e^2/h)T[G_{cd} - (2e^2/h)T]/[G_{ab} - (2e^2/h)T] \tag{7b}$$

$$G_3 = [G_{ab}G_{cd} - (2e^2/h)^2T^2]/[(G_{ab} + G_{cd} - 2(2e^2/h)T)]. \tag{7c}$$

For the pure ballistic case, with T defined by equation (6), we obtain for $G_{ab} < G_{cd}$, $G_1 = G_{ab}$, $G_2 = \infty$ and $G_3 = G_{ab}$ (or $R_1 = R_{ab}$, $R_2 = 0$ and $R_3 = R_{ab}$). For $G_{ab} > G_{cd}$ we obtain $G_1 = \infty$, $G_2 = G_{cd} = G_3$ (or $R_1 = 0$, $R_2 = R_3 = R_{cd}$). In the total absence of ballistic transport between B and D, $T = 0$ and $G_1 = G_{ab}$, $G_2 = G_{cd}$ and $G_3 = G_{ab}G_{cd}/(G_{ab} + G_{cd})$, or $R_1 = R_{ab}, R_2 = R_{cd}$ and the classical addition of resistance, $R_3 = R_1 + R_2$ is obtained.

For $G_{ab} = G_{cd}$, equation (7*c*) reduces to

$$G_3 = [G_{ab} + (2e^2/h)T]/2 \tag{8}$$

which is the result obtained by Beenakker and van Houten [5] for the series resistance of two identical QPCs.

Note that from equations (7*a*) and (7*b*)

$$(G_1 - G_{ab})(G_2 - G_{cd}) = (2e^2/h)^2 T^2. \tag{9}$$

We have calculated values of T from series of traces such as those shown in figure 3. For each set, the bias on gates a and b is held constant at some value corresponding to a quantised value of G_{ab}. The voltage on c and d is then varied between 0 and pinch-off. T has the value 0 for classical series resistance and is defined by equation (6) for the ballistic case. For direct comparison between different configurations we define the normalised transmission coefficient

$$\bar{T} = (2e^2/h)T/[\mathrm{Min}(G_{ab}, G_{cd})] \tag{10}$$

which has the property $\bar{T} = 0$ (classical) and $\bar{T} = 1$ (ballistic).

Values of T can be obtained using equation (7) or equation (9). We find that equations (7*c*) and (9) give far more consistent and reproducible results than equations (7*a*) and (7*b*). In almost all cases the values calculated from (7*c*) and (9) agree within experimental uncertainty. The interpretation of the data using equations (7*a*) and (7*b*) is not so successful. In some cases the measured values of G_1 and G_2 are 'impossible', i.e. it is not possible to fit equations (7*a*) or (7*b*) with any value of T.

The procedure for calculating T is as follows. For each set of traces (for example those shown in figure 3) we first identify the plateaux. G_{ab} is fixed and is therefore known so the plateaux correspond to the quantised resistance steps in G_{cd}. In figure 3 $G_{ab}^{-1} = 6493\ \Omega$ and we identify the plateaux as shown. We then take the value of G_{cd} to be the value of the quantised resistance corresponding to each plateau. Thus, for the first plateau we take $G_{cd}^{-1} = 12909\ \Omega$ etc. We do not expect this to introduce errors larger than $\sim 1\%$ in the value of G_{cd}. We assume that the variation of the gate bias on c and d has no effect on G_{ab}. This assumption is corroborated by the observation that the plateaux in figure 3 occur at the same values of the gate voltage on c and d and when the bias on a and b is zero.

In table 1 we show the values of $\bar{T}$ in terms of the conductances G_{ab} and G_{cd} expressed as the number of conducting 1D channels in each QPC. For example, if $N_{ab} = 2$ then $G_{ab} = 2(2e^2/h)$. We note two interesting points:

(i) the transmission coefficient is symmetric with respect to the two QPCs. For example, if $N_{ab} = 1$ and $N_{cd} = 4$, the transmission coefficient is the same as for $N_{ab} = 4$ and $N_{cd} = 1$. This illustrates the symmetry of these gate pairs and shows that the two QPCs are behaving in a very similar fashion.

(ii) $\bar{T}$ is always a minimum when $N_{ab} = N_{cd} = N_0$. This is illustrated in figure 4 where $N_{ab} = 3$. $\bar{T}$ is plotted against N_{cd}. There is a clear minimum for $N_{cd} = 3$. Similar behaviour is also shown in figure 4 for $N_{ab} = 4$ and occurs for all values of N. There is *no* strong systematic *variation* of $\bar{T}$ with N_0, though there may be a trend that $\bar{T}$ is smaller as N_0 becomes smaller. This appears to contradict the prediction of Beenakker and van

Table 1. The normalised transmission coefficient, $\bar{T}$, for various values of N_{ab} and N_{cd}.

N_{ab}	N_{cd}	$\bar{T}$ (error: ±5%)	N_{ab}	N_{cd}	$\bar{T}$ (error: ± 5%)
1	1	0.52	3	2	0.68
1	2	0.73	3	3	0.66
1	3	0.80	3	4	0.68
1	4	1.00	3	5	0.74
2	1	0.76	3	6	0.78
2	2	0.49	3	7	0.84
2	3	0.75	4	1	1.00
2	4	0.78	4	2	0.80
2	5	0.80	4	3	0.67
2	6	0.83	4	4	0.61
2	7	0.80	4	5	0.69
3	1	0.80	4	6	0.79

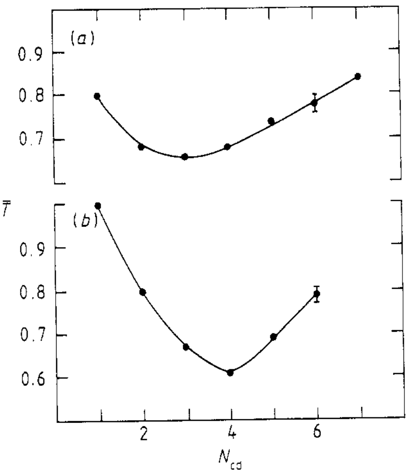

Figure 4. The normalised transmission coefficient, $\bar{T}$, plotted against N_{cd} with fixed (*a*) N_{ab} = 3 and (*b*) N_{ab} = 4. The lines are guides to the eye.

Houten [5] that $\bar{T}$ should increase, for a given gate geometry, as the channel becomes narrower. Also our numerical values for $\bar{T}$ in the case of $N_{ab} = N_{cd}$ are much lower than they predict for our geometry. For our sample geometry they predict T^- to be between 0.9 and 1 always, in contrast to the observed values.

To summarise, we have measured the resistance of two QPCs in series. We find that there is a substantial degree of adiabatic ballistic transport and by employing a simple model we are able to calculate a value for the ballistic transmission coefficient, $\bar{T}$, between the two contacts. This varies between 0.5 and 1.0 for our sample geometry. We are certain that the two QPCs remain distinct since we are able to measure the intermediate voltage between them. The minimum values of $\bar{T}$ always occur when the conductances of the two QPCs are equal. This result is intuitively plausible and consistent with a diffracting wave picture, but it is more difficult to interpret with ballistic trajectories.

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